

Exercise 21

For the following exercises, find the domain, range, and all zeros/intercepts, if any, of the functions.

$$g(x) = \sqrt{\frac{7}{x-5}}$$

Solution

Split up the square root function.

$$g(x) = \frac{\sqrt{7}}{\sqrt{x-5}}$$

The argument of a square root function has to be nonnegative; in addition, the denominator cannot be zero.

$$\begin{aligned}x-5 &\geq 0 & \text{and} & & x-5 &\neq 0 \\x &\geq 5 & \text{and} & & x &\neq 5\end{aligned}$$

Therefore, the domain is $\{x \mid x > 5\}$. $g(x)$ is continuous for $x > 5$, so it takes on all values between

$$g(5.0001) = \frac{\sqrt{7}}{\sqrt{5.0001-5}} \approx 265$$

$$g(50\,000) = \frac{\sqrt{7}}{\sqrt{50\,000-5}} \approx 0.012.$$

Choosing a value of x even closer to 5 gives an even bigger number, and choosing an even larger value for x yields a number closer to 0. The range is then $\{y \mid 0 < y < \infty\}$. Below is a graph of $g(x)$ versus x to confirm these results.

